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Static BPS black hole in $4d$ higher-spin gauge theory

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Abstract

We find exact spherically symmetric solution of $4d$ nonlinear bosonic higher-spin gauge theory, that preserves a quarter of supersymmetries of $\mathcal{N} = 2$ supersymmetric $4d$ higher-spin gauge theory. In the weak field regime it describes AdS_4 Schwarzschild black hole in the spin two sector along with non-zero massless fields of all integer spins.

1 Introduction

Nonlinear field equations of higher spin (HS) gauge fields were originally found for the $4d$ HS theory [1, 2]. Then these results were extended to $d = 3$ [3] and any d [4]. In $d \geq 4$, HS gauge theories describe interactions of infinite sets of propagating massless fields of lower and higher spins. The theory is manifestly general coordinate invariant, containing gravity as its part. In HS theory, spin two sources HS fields and vice versa. As a result, solutions of Einstein gravity are not necessarily solutions of the HS theory. HS interactions may significantly affect the theory in the strong field regime. Moreover, since the interval $ds^2 = g_{mn}dx^m dx^n$ associated to the $s = 2$ field g_{mn} is not invariant under HS gauge symmetry, standard concepts of general relativity have to be reconsidered in the HS theory with unbroken HS symmetries.

The difficulty of finding solutions in HS theory is due to nonlocality of the field equations in the auxiliary noncommutative twistor space, rendered by the star-product operation. By field equations this nonlocality in the twistor space is mapped to space-time nonlocality of the HS field equations at the interaction level in accordance with the fact that HS interactions contain higher derivatives [5, 6]. (For review and more references on HS gauge theories see [7].) As a result, the nonlinear HS gauge theory goes beyond the low energy limit typical for Einstein theory and its perturbative stringy corrections, having capacity to account strong field effects.

So far a very few exact solutions of the HS theory were available. The simplest one is the AdS_d vacuum solution. BTZ black hole (BH) [8] also solves $3d$ nonlinear HS field equations [9]. The first nontrivial example of exact solution of $3d$ HS theory was found in [10], where it was shown that for non-zero HS curvature vacuum field $B_0 = \nu$, the $3d$ HS field equations describe massive matter fields with the ν-dependent mass parameter. Some solutions of $4d$ HS equations were studied by Sezgin and Sundell [11, 12]. More general class of solutions with

non-vanishing HS fields, which reveal examples of algebraically special solutions of HS theory, was obtained by Iazeolla, Sezgin and Sundell in [13]. A large class of solutions of this type solves chiral models with $(0,4)$ or $(2,2)$ space-time signatures. In the noncommutative twistor sector they effectively amount to the $3d$ solution of [10], but their physical meaning is still not completely clear.

Since HS theory extends Einstein gravity, the natural question is what are counterparts of BH solutions in HS theory? In this paper we answer this question for the simplest case of a spherically symmetric charged BH. Namely, we present explicit coordinate free construction of the exact solution of the bosonic sector of $\mathcal{N}=2$ supersymmetric $4d$ HS theory of [2], characterized by a single free dimensionful parameter M (for a given cosmological constant) that determines a value of the BH electric charge e. In the weak field approximation, where the terms proportional to e^2 are neglected, the obtained solution reproduces Schwarzschild BH of mass M for $s=2$ along with BH spin-s massless fields found in [14]. Beyond this limit, the obtained solution differs from Schwarzschild BH being supersymmetric, *i.e.*, BPS. This in turn suggests that its electric charge is critical and hence nonzero.

Our construction is based on the unfolded formulation of Einstein BH developed in [15] where AdS_4 Killing symmetries play a distinguished role. The presented HS BH solution is constructed in terms of a Fock vacuum in the HS star–product algebra associated to a particular Killing symmetry of AdS_4. This Ansatz exhibits dramatic simplification of the perturbative analysis of the nonlinear HS equations, reducing the problem to $3d$ HS equations. The same time, it makes the obtained solution supersymmetric.

The paper is organized as follows. In Section 2 we recall the formulation of AdS_4 BH of [15]. Nonlinear $4d$ HS bosonic equations are recalled in Section 3. In Section 4 we explain how BH and its HS extension arises in the free HS theory. The exact HS BH solution of the nonlinear HS field equations is obtained in Section 5. Its symmetries are found in Section 6. Conclusions and perspectives are discussed in Section 7. Conventions are summarized in Appendix.

2 AdS_4 Schwarzschild black hole

As is well-known, the metric of a BH of mass M in AdS_4 admits the Kerr-Schild form[1] [16]

$$g_{mn}=\eta_{mn}+\frac{2M}{r}k_mk_n\,,\qquad g^{mn}=\eta^{mn}-\frac{2M}{r}k^mk^n\,,\tag{2.1}$$

where η_{mn}, $(m,n=0\dots3)$ is the background AdS_4 metric with negative cosmological constant $-\lambda^2$, k^m is the Kerr-Schild vector, that satisfies

$$k^mk_m=0\,,\quad k^m\mathcal{D}_mk_n=k^mD_mk_n=0\,,\tag{2.2}$$

where $\mathcal{D}_m$ and D_m are BH and AdS_4 covariant derivatives, respectively, and

$$\frac{1}{r}=-\frac{1}{2}\mathcal{D}_mk^m=-\frac{1}{2}D_mk^m\,.\tag{2.3}$$

Note, that it makes no difference in (2.1)-(2.3) whether indices are raised and lowered by either AdS_4 or full BH metrics. Although in this paper we proceed with the coordinate independent

[1] In this paper we set Newton constant $4\pi G=1$.

description, let us present explicit decomposition (2.1) in a coordinate system of [17] where k^m has a particularly simple form

$$\eta_{mn} = \frac{1}{1+\lambda^2 r^2} \begin{pmatrix} (1+\lambda^2 r^2)^2 & 0 & 0 & 0 \\ 0 & -1-\lambda^2(y^2+z^2) & \lambda^2 xy & \lambda^2 xz \\ 0 & \lambda^2 xy & -1-\lambda^2(x^2+z^2) & \lambda^2 yz \\ 0 & \lambda^2 xz & \lambda^2 yz & -1-\lambda^2(x^2+y^2) \end{pmatrix} \tag{2.4}$$

$$k^0 = \frac{1}{1+\lambda^2 r^2}\,, \quad k^1 = -\frac{x}{r}\,, \quad k^2 = -\frac{y}{r}\,, \quad k^3 = -\frac{z}{r}\,, \quad r^2 = x^2+y^2+z^2\,. \tag{2.5}$$

Let AdS_4 be described by the zero-curvature equation

$$R_{0AB} = dW_{0AB} + \frac{1}{2} W_{0A}{}^{C} \wedge W_{0CB} = 0\,, \qquad A, B = 1\ldots 4\,, \tag{2.6}$$

where $d = dx^n \frac{\partial}{\partial x^n}$ and $W_{0AB}(x) = W_{0BA}(x)$ is the $sp(4)$ connection 1-form. Indices $A, B, \ldots$ are raised and lowered by the $sp(4)$ symplectic form identified with $4d$ charge conjugation matrix (see Appendix). In two-component spinor notation

$$W_{0AB} = \begin{pmatrix} \omega_{\alpha\beta} & -\lambda h_{\alpha\dot\beta} \\ -\lambda h_{\beta\dot\alpha} & \bar\omega_{\dot\alpha\dot\beta} \end{pmatrix}, \quad \omega_{\alpha\beta} = \omega_{\beta\alpha}\,, \quad \bar\omega_{\dot\alpha\dot\beta} = \bar\omega_{\dot\beta\dot\alpha}\,, \tag{2.7}$$

where 1-forms $\omega_{\alpha\beta}$ and $h_{\alpha\dot\alpha}$ describe the Lorentz connection and vierbein, respectively.

AdS_4 possesses ten global symmetry parameters $K_{AB} = K_{BA}$ valued in the $o(3,2) \sim sp(4)$ Lie algebra, that satisfy

$$D_0 K_{AB} = 0\,, \qquad D_0^2 = 0\,, \tag{2.8}$$

where D_0 is the AdS_4 covariant differential (e.g., $D_0 A_A = dA_A + W_0{}_A{}^B A_B$) that squares to zero by virtue of (2.6). In terms of two-component spinors with

$$K_{AB} = \begin{pmatrix} \lambda^{-1}\varkappa_{\alpha\beta} & v_{\alpha\dot\beta} \\ v_{\beta\dot\alpha} & \lambda^{-1}\bar\varkappa_{\dot\alpha\dot\beta} \end{pmatrix}, \tag{2.9}$$

(2.8) amounts to

$$D^L v_{\alpha\dot\alpha} = \frac{1}{2} h^{\gamma}{}_{\dot\alpha}\varkappa_{\gamma\alpha} + \frac{1}{2} h_{\alpha}{}^{\dot\gamma}\bar\varkappa_{\dot\alpha\dot\gamma}\,, \qquad D^L\varkappa_{\alpha\alpha} = \lambda^2 h_\alpha{}^{\dot\gamma} v_{\alpha\dot\gamma}\,, \qquad D^L\bar\varkappa_{\dot\alpha\dot\alpha} = \lambda^2 h^{\gamma}{}_{\dot\alpha} v_{\gamma\dot\alpha}\,, \tag{2.10}$$

where

$$D^L A_\alpha = dA_\alpha + \frac{1}{2}\omega_\alpha{}^\gamma A_\gamma\,, \qquad D^L \bar A_{\dot\alpha} = d\bar A_{\dot\alpha} + \frac{1}{2}\bar\omega_{\dot\alpha}{}^{\dot\gamma}\bar A_{\dot\gamma}\,.$$

From (2.10) it follows in particular that $v_{\alpha\dot\alpha}$ is an AdS_4 Killing vector in the local frame.

According to [15], the Kerr–Schild vector of the BH metric (2.1) and its derivatives are expressed in terms of K_{AB} (2.9)

$$k_{\alpha\dot\alpha} \equiv h^n_{\alpha\dot\alpha} k_n = \frac{1}{v^- v^+} v^-_{\alpha\dot\alpha}\,, \qquad v^\pm_{\alpha\dot\alpha} = \pi^{\pm\gamma}_{\alpha}\bar\pi^{\pm\dot\gamma}_{\dot\alpha} v_{\gamma\dot\gamma}\,, \qquad v^- v^+ = \frac{1}{2} v^-_{\alpha\dot\alpha} v^{+\alpha\dot\alpha}\,, \tag{2.11}$$

where

$$\pi^\pm_{\alpha\beta} = \frac{1}{2}(\epsilon_{\alpha\beta} \pm \frac{\varkappa_{\alpha\beta}}{\sqrt{-\varkappa^2}})\,, \qquad \varkappa^2 = \frac{1}{2}\varkappa_{\alpha\beta}\varkappa^{\alpha\beta} \tag{2.12}$$

are projectors, that satisfy

$$\pi_\alpha^{\pm\gamma}\pi^{\pm}_{\gamma\beta} = \pi^{\pm}_{\alpha\beta}\,, \qquad \pi_\alpha^{\pm\gamma}\pi^{\mp}_{\gamma\beta} = 0\,. \tag{2.13}$$

(The conjugated projectors $\bar\pi^{\pm}_{\dot\gamma\dot\beta}$ satisfy analogous relations.) Note that, alternatively, one can use $v^{+}_{\alpha\dot\alpha}$ in (2.11) instead of $v^{-}_{\alpha\dot\alpha}$ [14].

A type of the BH (e.g., whether it is rotating or static) depends on the values of $sp(4)$ invariants associated to K^{AB}. The static case is characterized by the condition

$$K_A{}^B K_B{}^C = \delta_A{}^C \tag{2.14}$$

equivalent to

$$\lambda^{-2}\varkappa^2 + v^2 = -1\,, \quad \varkappa^2 = \bar\varkappa^2\,, \quad \bar\varkappa_{\dot\alpha}{}^{\dot\gamma} v_{\beta\dot\gamma} + v^{\gamma}{}_{\dot\alpha}\varkappa_{\gamma\beta} = 0\,. \tag{2.15}$$

The function r (2.3) satisfies [15]

$$\frac{1}{r} = \frac{\lambda^2}{\sqrt{-\varkappa^2}}\,, \qquad d\left(\frac{1}{r}\right) = \frac{1}{2\lambda^2 r^3} h^{\alpha\dot\alpha} v^{\alpha}{}_{\dot\alpha}\varkappa_{\alpha\alpha}\,.$$

Einstein equations for the metric (2.1) are satisfied as a consequence of (2.8) and (2.11).

The Kerr-Schild BH solution admits a *HS Kerr-Schild* generalization for massless bosonic fields of all spins [14]

$$\phi_{m_1\ldots m_k} = \frac{2M}{r} k_{m_1}\ldots k_{m_k} \tag{2.16}$$

that satisfy free spin-s equation in the AdS_4 background

$$D^n D_n \varphi_{m(s)} - s D_n D_m \varphi^n{}_{m(s-1)} = -2(s-1)(s+1)\lambda^2\varphi_{m(s)}\,. \tag{2.17}$$

The case of $s = 2$ reproduces Kerr-Schild term of the BH metric (2.1). As shown in Sections 4 and 5, the HS Kerr-Schild fields solve free field equations of the linearized HS theory and remain nonzero in the obtained HS BH solution.

3 HS equations in four dimensions

To reproduce Kerr-Schild solution in HS theory let us recall the structure of nonlinear $4d$ HS equations. (For more detail see [18].) Starting from this section we set $\lambda^2 = 1$ for convenience.

HS nonlinear equations in $d = 4$ are formulated in terms of 1-forms $W(Z,Y|x) = dx^n W_n(Z,Y|x)$ and 0-forms $B(Z,Y|x)$ that depend on space-time coordinates x^n and auxiliary spinor variables Z^A and Y^A. In addition, the 1-form connection along Z-direction should be introduced $S(Z,Y|x) = S_\alpha(Z,Y|x)dz^\alpha + \bar S_{\dot\alpha}(Z,Y|x)d\bar z^{\dot\alpha}$ to be expressed via dynamical fields by the field equations. It is required that $\{dx^n, dz^\alpha\} = 0$, $\{dx^n, d\bar z^{\dot\alpha}\} = 0$. In this section, we consider the bosonic sector of the HS equations of [2], where the fields B and W are even functions of the oscillators (Z,Y) and S is odd. The simplest version of HS equations with the topological fields factored out is

$$dW - W \star \wedge W = 0\,, \tag{3.1}$$

$$dB - W \star B + B \star \tilde W = 0\,, \tag{3.2}$$

$$dS_\alpha - [W, S_\alpha]_\star = 0\,, \quad d\bar S_{\dot\alpha} - [W, \bar S_{\dot\alpha}]_\star = 0\,, \tag{3.3}$$

$$S_\alpha \star S^\alpha = 2(1 + B \star v)\,, \quad \bar{S}_{\dot\alpha} \star \bar{S}^{\dot\alpha} = 2(1 + B \star \bar{v})\,, \quad [S_\alpha, \bar{S}_{\dot\alpha}]_\star = 0\,, \tag{3.4}$$

$$B \star \tilde{S}_\alpha + S_\alpha \star B = 0\,, \quad B \star \tilde{\bar{S}}_{\dot\alpha} + \bar{S}_{\dot\alpha} \star B = 0\,, \tag{3.5}$$

where $\tilde{A} = (-u_\alpha, \bar{u}_{\dot\alpha})$ for $A = (u_\alpha, \bar{u}_{\dot\alpha})$ and

$$v = \exp(z_\alpha y^\alpha)\,, \qquad \bar{v} = \exp(\bar{z}_{\dot\alpha} \bar{y}^{\dot\alpha})\,. \tag{3.6}$$

The star-product in the auxiliary space of commuting variables $Y_A = (y_\alpha, \bar{y}_{\dot\alpha})$, $Z_A = (z_\alpha, \bar{z}_{\dot\alpha})$ is defined by

$$(f \star g)(Z, Y) = \frac{1}{(2\pi)^8} \int d^4u d^4v f(Z+U, Y+U) g(Z-V, Y+V) e^{U_A V^A}\,, \quad U_A V^A = u_\alpha v^\alpha + \bar{u}_{\dot\alpha} \bar{v}^{\dot\alpha}\,. \tag{3.7}$$

An integration contour in (3.7) is chosen so that $1 \star f = f \star 1 = f$. Note that the star-product definition (3.7) differs from that of [18] by the absence of the imaginary unit factor in the exponential. From (3.7) it follows in particular

$$Y_A \star f = (Y_A - \frac{\partial}{\partial Y^A} + \frac{\partial}{\partial Z^A}) f\,, \quad f \star Y_A = (Y_A + \frac{\partial}{\partial Y^A} + \frac{\partial}{\partial Z^A}) f\,,$$

$$Z_A \star f = (Z_A - \frac{\partial}{\partial Y^A} + \frac{\partial}{\partial Z^A}) f\,, \quad f \star Z_A = (Z_A - \frac{\partial}{\partial Y^A} - \frac{\partial}{\partial Z^A}) f\,, \tag{3.8}$$

$$[z_\alpha, z_\beta]_\star = -[y_\alpha, y_\beta]_\star = 2\epsilon_{\alpha\beta}\,, \quad [\bar{z}_{\dot\alpha}, \bar{z}_{\dot\beta}]_\star = -[\bar{y}_{\dot\alpha}, \bar{y}_{\dot\beta}]_\star = 2\epsilon_{\dot\alpha\dot\beta}\,, \quad [y_\alpha, \bar{y}_{\dot\alpha}]_\star = [z_\alpha, \bar{z}_{\dot\alpha}]_\star = 0\,. \tag{3.9}$$

An important property of the star-product (3.7) is that it admits left and right inner Klein operators (3.6) that satisfy

$$v \star v = \bar{v} \star \bar{v} = 1\,, \qquad v \star f(z, y) = f(-z, -y) \star v\,, \qquad \bar{v} \star f(\bar{z}, \bar{y}) = f(-\bar{z}, -\bar{y}) \star \bar{v} \tag{3.10}$$

and

$$v \star f(z, y) = \exp(z_\alpha y^\alpha) f(y, z)\,, \qquad \bar{v} \star f(\bar{z}, \bar{y}) = \exp(\bar{z}_{\dot\alpha} \bar{y}^{\dot\alpha}) f(\bar{y}, \bar{z})\,. \tag{3.11}$$

Note, that Klein operators, that act only on Y or Z variables, are delta-functions [19] (hence not entire)

$$\delta(y) \star \delta(y) = 1\,, \qquad \delta(y) \star f(z, y) = f(z, -y) \star \delta(y)\,. \tag{3.12}$$

Also note that $\hat{f}(z, y) = f(z, y) \star \delta(y)$ gives Fourier transform with respect to y–variable $\hat{f}(z, y) = \int d^2u f(z, u) e^{-u_\alpha y^\alpha}$. Analogous formulae hold for dotted spinors and for $y \leftrightarrow z$.

It follows then that

$$v = \delta(y) \star \delta(z)\,, \qquad \bar{v} = \delta(\bar{y}) \star \delta(\bar{z})\,. \tag{3.13}$$

That the results are entire functions rather than distributions is because the star-product (3.7) is a specific normal star-product rather than Weyl star-product (for more detail see [18]).

The Eqs. (3.1)-(3.5) are consistent and manifestly invariant under the gauge transformations

$$\delta B = \epsilon \star B - B \star \tilde{\epsilon}\,, \quad \delta W = d\epsilon + [\epsilon, W]_\star\,, \quad \delta S_\alpha = [\epsilon, S_\alpha]_\star\,, \quad \delta \bar{S}_{\dot\alpha} = [\epsilon, \bar{S}_{\dot\alpha}]_\star \tag{3.14}$$

with an arbitrary gauge parameter $\epsilon = \epsilon(Z, Y|x)$. The vacuum solution for (3.1)-(3.5), that describes empty AdS_4 space, is $B_0 = 0\,,\quad S_0 = z_\alpha dz^\alpha + \bar{z}_{\dot\alpha} d\bar{z}^{\dot\alpha}\,,\quad W_0 = W_0(Y|x)\,,$ where

$$W_0(Y|x) = -\frac{1}{8}\Big(\omega_{\alpha\alpha}(x) y^\alpha y^\alpha + \bar{\omega}_{\dot\alpha\dot\alpha}(x) \bar{y}^{\dot\alpha} \bar{y}^{\dot\alpha} - 2h_{\alpha\dot\alpha}(x) y^\alpha \bar{y}^{\dot\alpha}\Big) \tag{3.15}$$

describes AdS_4 vacuum fields via Eq. (2.6) that acquires the form

$$\mathcal{D}_0^2 \equiv dW_0 - W_0 \star \wedge W_0 = 0\,. \tag{3.16}$$

The variables Z_A in the star-product (3.7), that play important role in the description of HS interactions, can be neglected at the free level. Following [20, 18] a free HS field is described in the unfolded formalism by the 1-forms $w(Y|x)$ and 0-forms $C(Y|x)$

$$w(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} w_{\alpha(n),\dot\alpha(m)} y^\alpha \dots y^\alpha \bar y^{\dot\alpha} \dots \bar y^{\dot\alpha}, \quad C(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{n!m!} C_{\alpha(n),\dot\alpha(m)} y^\alpha \dots y^\alpha \bar y^{\dot\alpha} \dots \bar y^{\dot\alpha} \tag{3.17}$$

that encode, respectively, linearized HS potentials and field strengths along with towers of auxiliary fields. $w(Y|x)$ and $C(Y|x)$ are the parts of $W(Z,Y|x)$ and $B(Z,Y|x)$, that remain unrestricted by the equations (3.3)-(3.5). In the sector of 0-forms, free equations resulting from (3.2) have a form of covariant constancy condition in the twisted–adjoint module

$$\tilde{\mathcal{D}}_0 C \equiv dC - W_0 \star C + C \star \tilde W_0 = 0\,, \qquad \tilde f(y,\bar y) = f(-y,\bar y)\,. \tag{3.18}$$

The linearized equations for HS gauge potentials, that result from (3.1), are [20]

$$R_{1\alpha(n),\dot\alpha(m)} = \delta(m) h^{\gamma\dot\beta} \wedge h^{\gamma}{}_{\dot\beta} C_{\alpha(n)\gamma(2)} + \delta(n) h^{\gamma\dot\beta} \wedge h_{\gamma}{}^{\dot\beta} C_{\dot\alpha(m)\dot\beta(2)} \tag{3.19}$$

($\delta(n) = 1(0)$ at $n = 0 (n \neq 0)$), where the curvatures $R_{1\alpha(n),\dot\alpha(m)}$ are components of the linearized HS curvature tensor

$$R_1(Y|x) \equiv \mathcal{D}_0 w(Y|x)\,. \tag{3.20}$$

Note, that Eq. (3.18) with a chosen W_0 is invariant under HS global symmetry transformation

$$\delta C = \epsilon_0 \star C - C \star \tilde\epsilon_0 \tag{3.21}$$

provided that

$$\mathcal{D}_0 \epsilon_0 = 0\,. \tag{3.22}$$

Since the twist operation in (3.18) can be realized as $\tilde f(Y) = \delta(y) \star f \star \delta(y)$, any solution of the global symmetry equation (3.22) $\epsilon_0(Y|x)$ generates a solution of (3.18) of the form

$$C(Y|x) = c_1 \epsilon_0(Y|x) \star \delta(y) + c_2 \epsilon_0(Y|x) \star \delta(\bar y)\,, \tag{3.23}$$

where c_1, c_2 are arbitrary constants. This formula manifests that adjoint and twisted adjoint covariant derivatives are related via Fourier transform of either y_α or $\bar y_{\dot\alpha}$ variables.

4 Black hole solution in free HS theory

As shown in [15], a generic AdS_4 BH is completely determined by a chosen global symmetry parameter K_{AB} of AdS_4. Let us apply this idea to a HS generalization. Since K_{AB} satisfies (2.8), it follows that

$$\mathcal{D}_0 f(K_{AB} Y^A Y^B) = 0 \tag{4.1}$$

for any $f(\xi)$. By (3.23), a solution of the free HS equation (3.18) is generated by

$$C(Y|x) = Mf(K_{AB}Y^AY^B) \star \delta(y)\,. \tag{4.2}$$

Generally, C (4.2) is not Hermitian yielding at the linearized level two different real solutions. In the $s = 2$ sector each can be shown to correspond to generic AdS_4–Kerr–NUT BH with mass $m \sim$ Re M and NUT charge $n \sim$ Im M for one and vice versa for another.

In this paper we confine ourselves to the simplest static case of (2.14) with real M and $f(\xi) = \exp(\xi/2)$,

$$F_K = \exp\,(\frac{1}{2}K_{AB}Y^AY^B)\,. \tag{4.3}$$

The coefficient 1/2 in the exponential (4.3) is chosen so that, due to (2.14), F_K satisfies

$$F_K \star F_K = F_K\,, \qquad F_K \star \delta(y) = F_K \star \delta(\bar{y})\,. \tag{4.4}$$

The first order HS BH curvature $C(Y|x) = MF_K \star \delta(y)$ has the form

$$C(Y|x) = \frac{M}{r}\exp\Big(\frac{1}{2}\varkappa^{-1}_{\alpha\beta}y^\alpha y^\beta + \frac{1}{2}\bar{\varkappa}^{-1}_{\dot{\alpha}\dot{\beta}}\bar{y}^{\dot{\alpha}}\bar{y}^{\dot{\beta}} - \varkappa^{-1}_{\alpha\gamma}v^\gamma{}_{\dot{\alpha}}y^\alpha\bar{y}^{\dot{\alpha}}\Big)\,, \tag{4.5}$$

where $\varkappa^{-1}_{\alpha\beta} = -\frac{1}{\varkappa^2}\varkappa_{\alpha\beta}$ and $r = \sqrt{-\varkappa^2}$. From (4.5) it follows that HS Weyl tensors are

$$C_{\alpha(2s)} = \frac{M}{2^s r}(\varkappa^{-1}_{\alpha\alpha})^s\,, \qquad \bar{C}_{\dot{\alpha}(2s)} = \frac{M}{2^s r}(\bar{\varkappa}^{-1}_{\dot{\alpha}\dot{\alpha}})^s\,. \tag{4.6}$$

According to [15], these are Petrov type–D Weyl tensors, that describe a Schwarzschild BH of mass M in the $s = 2$ sector along with a tower of Kerr-Schild fields (2.16) of all spins s. Note that the frame-like HS connections corresponding to (4.6), that carry equal numbers of undotted and dotted spinors, can be shown to be gauge equivalent to

$$W_{phys} = \frac{M}{r}h^{\alpha\dot{\alpha}}k_{\alpha\dot{\alpha}}\exp\Big(-\frac{1}{2}k_{\beta\dot{\beta}}y^\beta\bar{y}^{\dot{\beta}}\Big)\,. \tag{4.7}$$

5 Black hole in nonlinear HS theory

Starting with Schwarzschild solution (4.5) at the free level, we have to analyze higher-order corrections within the system (3.1)-(3.5). The specific choice (4.3) simplifies the analysis, allowing us to solve the problem exactly.

To reconstruct the HS field strength $B(Z, Y|x)$ and the 1-form connection $S(Z, Y|x)$ along Z directions one has first to solve the constraints (3.4), (3.5) which form a closed subsystem. This fixes dynamics completely reducing the problem to determination of HS potentials in terms of their curvatures via the equations (3.1)-(3.3).

5.1 BH Fock vacua

The key fact is that F_K (4.3) generates a Fock vacuum of the star-product algebra, defining a subalgebra of a reduced set of oscillators. Indeed, let us introduce the projectors $\Pi_{\pm AB}$

$$\Pi_{\pm AB} = \frac{1}{2}(\epsilon_{AB} \pm K_{AB})\,, \qquad \Pi_{\pm A}{}^C\Pi_{\pm C}{}^B = \Pi_{\pm A}{}^B\,, \quad \Pi_{\pm A}{}^C\Pi_{\mp C}{}^B = 0\,. \tag{5.1}$$

The creation and annihilation operators $Y_{\pm A} = \Pi_{\pm A}{}^B Y_B$ satisfy

$$[Y_{+A}, Y_{+B}]_\star = [Y_{-A}, Y_{-B}]_\star = 0\,, \qquad [Y_{+A}, Y_{-B}]_\star = \Pi_{+AB}\,, \tag{5.2}$$

$$Y_{-A} \star F_K = F_K \star Y_{+A} = 0\,. \tag{5.3}$$

In the nonlinear case, we have to analyze corrections due to Z–dependence of the HS curvature and connection. F_K is the only Z–independent element of the star-product algebra (3.7) that satisfies (5.3). More generally, let F be a space of elements $f(Z,Y|x)$ that satisfy

$$Y_{-A} \star f = f \star Y_{+A} = 0\ : \qquad f = F_K \phi(A|x)\,, \tag{5.4}$$

where ϕ is an arbitrary function and

$$A_A = (a_\alpha, \bar{a}_{\dot\alpha}) = Y_{+A} + Z_{+A} - (Y_{-A} - Z_{-A}) = Z_A + K_A{}^B Y_B\,, \qquad [A_A, A_B]_\star = 4\epsilon_{AB}\,. \tag{5.5}$$

F is a subalgebra of the star–product algebra. Namely,

$$\Big(F_K \phi_1\Big) \star \Big(F_K \phi_2\Big) = F_K(\phi_1 * \phi_2)\,, \tag{5.6}$$

where we have introduced the induced star–product $*$ on the space of functions $\phi(A|x)$, that has the form

$$(\phi_1 * \phi_2)(A) = \int d^4U \phi_1(A + 2U_+)\phi_2(A - 2U_-)e^{2U_{+A}U_-^A}\,. \tag{5.7}$$

The integral (5.7) is normalized in such a way that $1 * \phi = \phi * 1 = \phi$. Star–product (5.7) is associative and describes the normal ordering of the operators $(Y_{-A} - Z_{-A})$ and $(Y_{+A} + Z_{+A})$.

Note that any function of the form $\tilde{F}_K = f(Z|x) \star F_K$ satisfies (5.3) and, hence, can be represented in the form (5.4). Indeed, using (3.7), one can easily check that

$$F_K \star f(Z|x) = \frac{1}{4} F_K \int d^2v d^2\bar{v} f(A - V|x) e^{\frac{1}{2}K_{AB}V^A V^B}\,. \tag{5.8}$$

For (anti)holomorphic functions, the integration can be performed further

$$F_K \star f(z) = \frac{1}{2r} F_K \int d^2v f(a - v) e^{\frac{1}{2}\varkappa^{-1}_{\alpha\beta} v^\alpha v^\beta}\,, \quad F_K \star f(\bar{z}) = \frac{1}{2r} F_K \int d^2\bar{v} f(\bar{a} - \bar{v}) e^{\frac{1}{2}\bar{\varkappa}^{-1}_{\dot\alpha\dot\beta} \bar{v}^{\dot\alpha} \bar{v}^{\dot\beta}}\,. \tag{5.9}$$

In the sequel we will work with (anti)holomorphic functions of $(\bar{a}_{\dot\alpha})a_\alpha$, using relations

$$[a_\alpha, f(a)]_* = 2\frac{\partial}{\partial a^\alpha} f(a)\,, \qquad \{a_\alpha, f(a)\}_* = 2(a_\alpha + \varkappa_\alpha{}^\beta \frac{\partial}{\partial a^\beta}) f(a)\,, \qquad [a_\alpha, \bar{a}_{\dot\alpha}]_* = 0\,. \tag{5.10}$$

The star–product (5.7) possesses the Klein operators $\mathcal{K}$ and $\bar{\mathcal{K}}$,

$$\mathcal{K} = \frac{1}{r} \exp\Big(\frac{1}{2}\varkappa^{-1}_{\alpha\alpha} a^\alpha a^\alpha\Big)\,, \qquad \bar{\mathcal{K}} = \frac{1}{r} \exp\Big(\frac{1}{2}\bar{\varkappa}^{-1}_{\dot\alpha\dot\alpha} \bar{a}^{\dot\alpha} \bar{a}^{\dot\alpha}\Big) \tag{5.11}$$

that satisfy

$$\mathcal{K} * \mathcal{K} = \bar{\mathcal{K}} * \bar{\mathcal{K}} = 1\,, \quad \{\mathcal{K}, a_\alpha\}_* = \{\bar{\mathcal{K}}, \bar{a}_{\dot\alpha}\}_* = 0\,, \quad [\mathcal{K}, \bar{\mathcal{K}}]_* = [\mathcal{K}, \bar{a}_{\dot\alpha}]_* = [\bar{\mathcal{K}}, a_\alpha]_* = 0 \tag{5.12}$$

and result from

$$F_K \star \delta(z) = F_K \mathcal{K}\,, \qquad F_K \star \delta(\bar{z}) = F_K \bar{\mathcal{K}}\,. \tag{5.13}$$

5.2 Ansatz and final result

The key observation is that the Eqs. (3.4), (3.5) can be solved exactly by the Ansatz

$$B = MF_K \star \delta(y)\,, \tag{5.14}$$

$$S_\alpha = z_\alpha + F_K \sigma_\alpha(a|x)\,, \qquad \bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + F_K \bar{\sigma}_{\dot\alpha}(\bar{a}|x) \tag{5.15}$$

with some functions $\sigma_\alpha(a|x)$, $\bar{\sigma}_{\dot\alpha}(\bar{a}|x)$ to be specified later. Within the induced star–product (5.7), this Ansatz reduces (3.4), (3.5) to two copies of $3d$ (anti)holomorphic *deformed oscillators* considered in [21, 10] in the context of $3d$ HS theories. Indeed, introducing

$$s_\alpha = a_\alpha + \sigma_\alpha(a|x)\,, \qquad \bar{s}_{\dot\alpha} = \bar{a}_{\dot\alpha} + \bar{\sigma}_{\dot\alpha}(\bar{a}|x) \tag{5.16}$$

and using (3.13), (4.4), (5.10), (5.12) and (5.13) one arrives at the following system

$$[s_\alpha, s_\beta]_* = 2\epsilon_{\alpha\beta}(1 + M\mathcal{K})\,, \qquad \{\mathcal{K}, s_\alpha\}_* = 0\,, \qquad \mathcal{K} * \mathcal{K} = 1\,. \tag{5.17}$$

The deformed oscillators (5.17) were originally discovered by Wigner in [22] in a somewhat different form. The BH mass M plays a role of the deformation parameter.

Analogous equations hold in the dotted sector

$$[\bar{s}_{\dot\alpha}, \bar{s}_{\dot\beta}]_* = 2\epsilon_{\dot\alpha\dot\beta}(1 + M\bar{\mathcal{K}})\,, \qquad \{\bar{s}_{\dot\alpha}, \bar{\mathcal{K}}\}_* = 0\,, \qquad \bar{\mathcal{K}} * \bar{\mathcal{K}} = 1\,. \tag{5.18}$$

In addition,

$$[s_\alpha, \bar{s}_{\dot\alpha}]_* = 0\,, \quad [s_\alpha, \bar{\mathcal{K}}]_* = 0\,, \quad [\bar{s}_{\dot\alpha}, \mathcal{K}]_* = 0\,, \quad [\mathcal{K}, \bar{\mathcal{K}}]_* = 0\,. \tag{5.19}$$

Eqs. (5.17)-(5.19) follow from (5.12) and (5.10).

A proper Ansatz for HS connection 1-form $W(Y, Z|x)$ is

$$W = W_0 + F_K\Big(\omega(a|x) + \bar{\omega}(\bar{a}|x)\Big)\,, \tag{5.20}$$

where $W_0(Y|x)$ is the vacuum connection (3.15) and the additional terms manifest holomorphic factorization with respect to the variables $(a_\alpha, \bar{a}_{\dot\alpha})$. Note, that (5.20) has no definite holomorphy properties in the $(y_\alpha, \bar{y}_{\dot\alpha})$ variables because both a_α and $\bar{a}_{\dot\alpha}$ mix y_α with $\bar{y}_{\dot\alpha}$ via (5.5).

From (3.15) and (3.8) it follows that

$$\mathcal{D}_0\big(F_K f(A|x)\big) = F_K\Big(\hat{d} - \frac{1}{2} dK^{AB} \frac{\partial^2}{\partial A^A \partial A^B}\Big) f(A|x)\,, \tag{5.21}$$

where the x–dependence of A_A has been taken into account with the aid of (2.8) and (3.16) so that the differential $\hat{d}$ in (5.21) only accounts the manifest x dependence, *i.e.,* $\hat{d}A = 0$. Using (5.21), the HS equations that remain to be solved reduce to

$$[s_\alpha, s_\beta]_* = 2\epsilon_{\alpha\beta}(1 + M\mathcal{K})\,, \tag{5.22}$$

$$\mathcal{Q}s_\alpha - [\omega, s_\alpha]_* = 0\,, \tag{5.23}$$

$$\mathcal{Q}\omega - \omega * \wedge \omega = 0\,, \tag{5.24}$$

and their complex conjugated, where

$$\mathcal{Q} = \hat{d} - \frac{1}{2} d\varkappa^{\alpha\alpha} \frac{\partial^2}{\partial a^\alpha \partial a^\alpha}\,. \tag{5.25}$$

The operator $\mathcal{Q}$ has the following properties inherited from $\mathcal{D}_0$

$$\mathcal{Q}\big(f(a|x) * g(a|x)\big) = \mathcal{Q}f(a|x) * g(a|x) + f(a|x) * \mathcal{Q}g(a|x)\,, \tag{5.26}$$

$$\mathcal{Q}^2 = 0\,, \qquad \mathcal{Q}a_\alpha = 0\,, \qquad \mathcal{Q}\mathcal{K} = 0\,. \tag{5.27}$$

Remarkably, despite $\mathcal{Q}$ (5.25) contains second derivatives in the oscillators, it respects the chain rule (5.26). This is because the star-product (5.7) is x-dependent, so that, acting on the star-product $*$, $\hat{d}$ effectively compensates the terms that would spoil (5.26). Note that a similar construction was recently discussed in [23] in a different context of the definition of the ring of solutions of unfolded HS field equations (see Appendix of [23]).

Since we consider purely bosonic problem, being an even function of a-oscillators, the connection (5.20) commutes to the Klein operators (5.11) and therefore solves (3.2). (In presence of fermions this would not be true.) In Section (5.3) we solve the equations (5.22)-(5.24) obtaining the following final result for HS BH

$$S_\alpha = z_\alpha + MF_K\frac{a^+_\alpha}{r}\int_0^1 dt\, \exp\Big(\frac{t}{2}\varkappa^{-1}_{\beta\beta}a^\beta a^\beta\Big)\,, \tag{5.28}$$

$$\bar{S}_{\dot\alpha} = \bar{z}_{\dot\alpha} + MF_K\frac{\bar{a}^+_{\dot\alpha}}{r}\int_0^1 dt\, \exp\Big(\frac{t}{2}\bar{\varkappa}^{-1}_{\dot\beta\dot\beta}\bar{a}^{\dot\beta}\bar{a}^{\dot\beta}\Big)\,, \tag{5.29}$$

$$B = \frac{M}{r}\exp\Big(\frac{1}{2}\varkappa^{-1}_{\alpha\beta}y^\alpha y^\beta + \frac{1}{2}\bar{\varkappa}^{-1}_{\dot\alpha\dot\beta}\bar{y}^{\dot\alpha}\bar{y}^{\dot\beta} - \varkappa^{-1}_{\alpha\gamma}v^\gamma{}_{\dot\alpha}y^\alpha\bar{y}^{\dot\alpha}\Big)\,, \tag{5.30}$$

$$W = W_0 + \frac{M}{8r}F_K d\tau^{\alpha\alpha}a^+_\alpha a^+_\alpha\int_0^1 dt\,(1-t)\exp\Big(\frac{t}{2}\varkappa^{-1}_{\beta\beta}a^\beta a^\beta\Big) + c.c. + F_K\mathbf{f}_0\,, \tag{5.31}$$

where

$$\tau_{\alpha\alpha} \equiv \frac{\varkappa_{\alpha\alpha}}{r}\,, \tag{5.32}$$

$$\mathbf{f}_0 = -\frac{M}{8}(\tau_{\alpha\alpha}\omega^{\alpha\alpha} + \bar{\tau}_{\dot\alpha\dot\alpha}\bar{\omega}^{\dot\alpha\dot\alpha}) + \frac{M}{4r}h^{\alpha\dot\alpha}(v_{\alpha\dot\alpha} + k_{\alpha\dot\alpha}) \tag{5.33}$$

and the Kerr-Schild vector $k_{\alpha\dot\alpha}$ is defined in (2.11). Let us stress, that it is the Ansatz (5.20), that effectively implied holomorphic factorization of the oscillators a_α and $\bar{a}_{\dot\alpha}$, allowed us to integrate the equations on HS connection. Note also, that (5.28) and (5.29) do not correspond to the standard gauge choice of the HS perturbative analysis of [2, 18] with $S(Z,Y)|_{Z=0} = 0$. Hence, to reproduce HS Kerr-Schild solution (4.7) at first order in M one has to apply an appropriate HS gauge transformation. In the first order, such a gauge transformation is $W^{can} = W + \mathcal{D}_0 g$, where $g = -\frac{1}{2}\int_0^1 dt z^\alpha S_\alpha|_{z\to tz} + c.c. + g_0(Y|x)$ with an arbitrary $g_0(Y|x)$.

5.3 Details of analysis

HS constraints (3.4), (3.5) have been reduced to Eq. (5.22). By analogy with the standard perturbative analysis [2, 18], in the first order in M this gives $\frac{\partial}{\partial a^\alpha}\sigma^\alpha = M\mathcal{K}$, that can be solved in the form

$$\sigma^\pm_\alpha(a|x) = \frac{M}{r}a^\pm_\alpha\int_0^1 dt\, \exp\Big(\frac{t}{2}\varkappa^{-1}_{\beta\beta}a^\beta a^\beta\Big)\,, \qquad a^\pm_\alpha \equiv \pi^{\pm\beta}_\alpha a_\beta\,, \tag{5.34}$$

where $\pi^{\pm}_{\alpha\beta}$ are projectors (2.12). This solves (5.22) exactly because $[\sigma^{\pm}_{\alpha}, \sigma^{\pm}_{\beta}]_* = 0$. Indeed, the projectors (2.12) make the antisymmetric matrix $[\sigma^{\pm}_{\alpha}, \sigma^{\pm}_{\beta}]_*$ one-dimensional and hence zero. Choosing for definiteness the plus sign in (5.34) we obtain (5.28)-(5.30).

An important property of (5.34) is that

$$\mathcal{Q}\sigma_\alpha = -\frac{1}{4}\frac{\partial}{\partial a^\alpha}\Omega^{\beta\beta}\{a_\beta, \sigma_\beta\}_* , \tag{5.35}$$

where $\Omega_{\alpha\alpha}(x)$ is the $sp(2)$ flat connection (except may be $r = 0$, cf. (5.32))

$$\Omega_{\alpha\alpha} = d\tau_\alpha{}^\gamma \tau_{\gamma\alpha} , \qquad d\Omega_{\alpha\alpha} - \Omega_\alpha{}^\gamma \wedge \Omega_{\gamma\alpha} = 0 . \tag{5.36}$$

To find HS connection 1-form $W(Y, Z|x)$ corresponding to (5.28)-(5.30) we start with the equation (5.23). Let us solve it perturbatively. In the first order in M we have

$$\frac{\partial \omega}{\partial a^\alpha} = -\frac{1}{2}\mathcal{Q}\sigma_\alpha . \tag{5.37}$$

Using (5.35), the first-order HS connection can be written in the following remarkable form

$$\omega(a|x) = \frac{1}{8}\Omega^{\alpha\alpha}\{a_\alpha, \sigma_\alpha\}_* + f_0 + O(M^2) , \tag{5.38}$$

where, $f_0(x)$ is some a_α–independent 1-form.

The observation (5.35), (5.38) suggests the exact solution of (5.23). Indeed, one may note that the first term in (5.38) is the linearized part of $\frac{1}{8}\Omega^{\alpha\alpha}(s_\alpha * s_\alpha - a_\alpha * a_\alpha)$. Using that bilinears of the deformed oscillators generate their $sp(2)$ rotations,

$$T_{\alpha\alpha} = s_\alpha * s_\alpha , \qquad [T_{\alpha\alpha}, s_\beta]_* = 4\epsilon_{\alpha\beta} s_\alpha , \tag{5.39}$$

we obtain the exact solution of (5.23) in the following simple form

$$\omega(a|x) = \frac{1}{8}\Omega^{\alpha\alpha}(s_\alpha * s_\alpha - a_\alpha * a_\alpha) + f_0 , \quad \bar{\omega}(\bar{a}|x) = \frac{1}{8}\bar{\Omega}^{\dot{\alpha}\dot{\alpha}}(\bar{s}_{\dot{\alpha}} * \bar{s}_{\dot{\alpha}} - \bar{a}_{\dot{\alpha}} * \bar{a}_{\dot{\alpha}}) + \bar{f}_0 . \tag{5.40}$$

Remarkably, the connection (5.40) in fact does not contain the $O(M^2)$ terms, *i.e.,*

$$\omega_2(a|x) = \frac{1}{8}\Omega^{\alpha\alpha}\sigma_\alpha * \sigma_\alpha = 0 . \tag{5.41}$$

The simplest way to see this is to observe that from (5.23) it follows $\frac{\partial \omega_2}{\partial a^\alpha} = \frac{1}{2}[\omega_1, \sigma_\alpha]_*$ and hence, $\pi^{-\gamma}_{\alpha}\frac{\partial}{\partial a^\gamma}\omega_2(a|x) = 0$, so that $\omega_2 = \omega_2(a^-|x)$. On the other hand, it is easy to see that $\omega_2(a|x)$ (5.41) is an entire function of the oscillators $a^\pm$ such that

$$(a^+\frac{\partial}{\partial a^+} - a^-\frac{\partial}{\partial a^-})\omega_2(a^\pm|x) = 2\omega_2(a^\pm|x) . \tag{5.42}$$

Hence, it should be zero. Note that the straightforward verification of this fact, that was also completed, is not trivial, implying interesting integral identities.

Having solved (3.3), it remains to verify the HS zero-curvature equation (5.24) to determine the HS connection completely. Plugging (5.40) into (5.24) and noting that

$$d\Omega^{\alpha\alpha}\{a_\alpha, \sigma_\alpha\}_* = -M d\Omega^{\alpha\alpha}\tau_{\alpha\alpha} \tag{5.43}$$

one finds

$$df_0 = \frac{M}{16} d\tau^{\alpha\gamma} \wedge d\tau_\gamma{}^\alpha \tau_{\alpha\alpha} \,. \tag{5.44}$$

Note, that the r.h.s. of (5.44) is consistent with $d^2 f_0 = 0$. Indeed, $d^2 f_0 = \frac{M}{16} d\tau^{\alpha\gamma} \wedge d\tau_\gamma{}^\alpha \wedge d\tau_{\alpha\alpha}$ which is a $3d$ volume 3-form for the "frame" $E_{\alpha\alpha} = d\tau_{\alpha\alpha}$, that is however zero since $\tau^{\alpha\alpha}\tau_{\alpha\alpha} = const$. Note that f_0 contributes to the HS connection (5.20) via its real combination $\mathbf{f}_0 = f_0 + \bar{f}_0$ that can be chosen in the form (5.33). It is straightforward to verify that (5.33) satisfies (5.44) plus its complex conjugated. Thus, the final result for HS connection is (5.31).

6 Symmetries

Global symmetries of a $4d$ static BH include $SO(3)$ spatial rotations and R^1 time translations both in asymptotically Minkowski and in AdS_4 (in fact, its universal covering) geometry. Infinitesimally, they form algebra $su(2) \oplus gl(1)$. A static Reissner–Nordström BH is in addition characterized by its electric charge e, reproducing Schwarzschild BH at $e = 0$. The critical value of charge $e^2 = M^2$, that corresponds to the extremal BH, is characterized by the coincidence of two BH horizons [24] and by SUSY, being BPS [25]. Note that, at the free field level, the $s = 1$ field in (5.30) is just the Maxwell field strength of the AdS_4 Reissner–Nordström potential.

In this section we analyze global symmetries of the obtained static HS BH solution, showing in particular that (i) its space-time symmetry is $su(2) \oplus gl(1)$, (ii) it is supersymmetric, preserving a quarter of $4d$ $\mathcal{N} = 2$ SUSY of the nonlinear HS model of [2], and (iii) it possesses infinite dimensional HS extension of (super)symmetries of (i) and (ii).

6.1 Bosonic symmetries

From (3.14) it follows that the leftover global symmetry parameter $\epsilon_0(Y|x)$ should satisfy $\epsilon_0 \star B - B \star \tilde{\epsilon}_0 = 0$. (Note that all symmetries with Z-dependent parameters $\epsilon_0(Z, Y|x)$ are spontaneously broken because of the Z-dependent vacuum part of S.) Taking into account (5.14), this gives

$$\epsilon_0 \star F_K - F_K \star \epsilon_0 = 0 \,. \tag{6.1}$$

Since the Fock vacuum F_K satisfies (5.3), the general solution of (6.1) is

$$\epsilon_0(Y|x) = \sum_{m,n=1}^{\infty} f_{0A(m),B(n)}(x) \underbrace{Y_+^A \star \ldots \star Y_+^A}_{m} \star \underbrace{Y_-^B \star \ldots \star Y_-^B}_{n} + c_0(x) \,. \tag{6.2}$$

Now we observe that any $\epsilon_0(Y|x)$ (6.2) commutes to any $F_K \phi(A|x)$ as one can see from Eq. (5.4). As a result, the only nontrivial condition in (3.14) that remains is $d\epsilon_0 + [\epsilon_0, W_0]_\star = 0$. This requires $\epsilon_0(Y|x)$ (6.2) (*i.e.*, $f_{0A(m),B(n)}(x)$ and $c_0(x)$) be AdS_4 covariantly constant.

A maximal finite dimensional subalgebra of (6.2) is spanned by the bilinears of Y_- and Y_+ and constants. In particular, it contains generators of $su(2) \oplus gl(1)$

$$T^{AB} = Y_+^{(A} Y_-^{B)} \,, \qquad T = Y_{-A} Y_+^A \tag{6.3}$$

that belong to the class (6.2). Since the algebra $sp(4)$ of AdS_4 space-time symmetries is spanned by various bilinears of Y_A, this $su(2) \oplus gl(1)$ describes space-time symmetries that remain

unbroken. Hence, the obtained solution indeed describes a spherically symmetric static BH. Note that the Y-independent constant parameter in (6.2) describes a $u(1)$ inner symmetry. The full set of parameters (6.2) describes an infinite dimensional HS algebra of global symmetries of the HS BH solution.

6.2 Supersymmetry

The solution (5.28)-(5.31) turns out to be supersymmetric, which is most easily seen from the embedding of the bosonic HS equations considered so far into the $\mathcal{N}=2$ supesymmetric nonlinear HS system of [2] (see also [18]) which has the form

$$d\mathcal{W} - W \star \wedge \mathcal{W} = 0\,, \qquad d\mathcal{B} - [\mathcal{W}, \mathcal{B}]_\star = 0\,, \qquad d\mathcal{S} - [\mathcal{W}, \mathcal{S}]_\star = 0\,, \tag{6.4}$$

$$\mathcal{S} \star \mathcal{S} = dz_\alpha dz^\alpha (1 + \mathcal{B} \star kv) + d\bar{z}_{\dot\alpha} d\bar{z}^{\dot\alpha} (1 + \mathcal{B} \star \bar{k}\bar{v})\,, \qquad [\mathcal{S}, \mathcal{B}]_\star = 0\,, \tag{6.5}$$

where $\mathcal{W} = \mathcal{W}(Z, Y; k, \bar{k}|x)$, $\mathcal{B} = \mathcal{B}(Z, Y; k, \bar{k}|x)$ and $k, \bar{k}$ are the exterior Klein operators that satisfy $k^2 = \bar{k}^2 = 1$, $[k, \bar{k}] = [k, dx^\mu] = [\bar{k}, dx^\mu] = 0$ and

$$kf(Z, Y; dZ; k, \bar{k}|x) = f(\tilde{Z}, \tilde{Y}; d\tilde{Z}; k, \bar{k}|x)k\,, \quad \bar{k}f(Z, Y; dZ; k, \bar{k}|x) = f(-\tilde{Z}, -\tilde{Y}; -d\tilde{Z}; k, \bar{k}|x)\bar{k}\,, \tag{6.6}$$

where $\tilde{U}_A = (-u_\alpha, \bar{u}_{\dot\alpha})$ for $U_A = (u_\alpha, u_{\dot\alpha})$.

It is important to note that the system (6.4), (6.5) describes the doubled set of massless fields where all fields of integer and half-integer spins appear in two copies. Its bosonic sector consists of two independent subsystems described by the equations (3.1)-(3.5), which can be projected from the system (6.4), (6.5) by the projectors

$$P^\pm = \frac{1}{2}(1 \pm k\bar{k})\,, \qquad P^\pm P^\pm = P^\pm\,, \qquad P^\mp P^\pm = 0\,, \tag{6.7}$$

that by (6.6) commute to boson fields, that are even in spinorial variables, but not to fermions. As such, the bosonic reduction of the system (6.4), (6.5) describes two parallel bosonic worlds. Each of them is not supersymmetric because $P^\pm$ do not commute to fermions. Conventional $\mathcal{N}=2$ SUSY is achieved in the "diagonal world" described by the metric $g^{nm} = \frac{1}{2}(g^{+nm} + g^{-nm})$.

A natural BH solution of the supersymmetric HS theory is a combination of BH solutions in each of the bosonic sectors, where, *a priori*, one can choose solutions with unrelated $K^\pm_{AB}$. We wish however to consider the case where $K^+_{AB} = -K^-_{AB}$ and the corresponding Fock vacua (4.3) $F^\pm_K$ have opposite properties[2] (5.3)

$$Y_{\mp A} F^\pm_K = F^\pm_K Y_{\pm A} = 0\,, \qquad F^\pm_K = \exp\Big(\pm \frac{1}{2} K_{AB} Y^A Y^B\Big)\,. \tag{6.8}$$

Consider the bosonic solution of the $\mathcal{N}=2$ supersymmetric HS system of the form

$$\mathcal{W} = (P^+ W^+ + P^- W^-)\,, \quad \mathcal{B} = \frac{1}{2}(B^+(k + \bar{k}) + iB^-(k - \bar{k}))\,, \quad \mathcal{S} = (P^+ S^+ + P^- S^-)\,. \tag{6.9}$$

(Note that the factor of i in the definition of $\mathcal{B}$ in Eq.(6.5) is enforced by the reality conditions that conjugate k and $\bar{k}$ [2].) The labels $\pm$ refer to the solutions in the sectors of $P^\pm$ that may

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[2] Let us note that although the Fock vacua $F^\pm_K$ have ill-defined (infinite) mutual star–product this does not matter because, living in "parallel worlds" (*i.e.,* being multiplied by $P^\pm$), they never meet.

have different masses $M^{\pm}$. It is however convenient to demand the vacuum fields to coincide: $W_0^{\pm} = W_0$, $S_0^{\pm} = S_0$. In the case where M^+ or M^- vanishes, the respective world is AdS_4.

A global symmetry parameter $\epsilon(Y;k\bar{k}|x)$ now depends on $k\bar{k} = P^+ - P^-$ and should satisfy the conditions

$$[\mathcal{B}, \epsilon]_\star = 0\,, \qquad [\epsilon, \mathcal{S}]_\star = 0\,, \qquad d\epsilon - [\mathcal{W}, \epsilon]_\star = 0\,. \tag{6.10}$$

These are verified by AdS_4 covariantly constant ϵ that admit the following two representations

$$\epsilon(Y;k\bar{k}|x) = \epsilon^+_{lA}(Y)\star Y_-^A P^+ + \epsilon^-_{lA}(Y)\star Y_+^A P^- + c_0 = P^+ Y_+^A \star \epsilon^+_{rA}(Y) + P^- Y_-^A \star \epsilon^-_{rA}(Y) + c_0 \tag{6.11}$$

with some $\epsilon^{\pm}_{lA}(Y)$ and $\epsilon^{\pm}_{rA}(Y)$. Indeed, for instance, the terms proportional to F^+P^+ are annihilated by Y_- from the left, Y_+ from the right and P^- from the both sides.

Clearly, the bosonic global symmetry parameters (6.2) belong to the class (6.11). However, now the parameters that are odd in Y_A are also allowed. In particular, global SUSY with an AdS_4 covariantly constant spinor parameter $\epsilon_-^A(x)$

$$\epsilon(Y;k\bar{k}|x) = \epsilon_-^A(x) P^- Y_{-A} = \epsilon_-^A(x) Y_{-A} P^+\,, \qquad D_0 \epsilon_-^A(x) = 0\,, \qquad \Pi^A_{+B}(x)\epsilon_-^B(x) = 0 \tag{6.12}$$

belongs to this class (recall that Y_A anticommute to $k\bar{k}$). This global SUSY is a quarter of the $\mathcal{N} = 2$ SUSY with the supergenerators $Q^1_A = Y_A$ and $Q^2_A = ik\bar{k}Y_A$ [26] of the AdS_4 vacuum of the system (6.4), (6.5). Hence, the HS BH subalgebra corresponds to a $\frac{1}{4}$ BPS state. This strongly indicates that the obtained HS BH solution should be extremal.

Covariantly constant solutions of (6.11) form an infinite dimensional HS superalgebra of global symmetries of the obtained BH solution.

7 Conclusion

The new exact solution of $4d$ bosonic HS theory announced in this paper is a HS generalization of a static BH in GR. At the free field level it contains Schwarzschild BH in the spin two sector. However, the contribution of HS fields is important in the strong field regime as is demonstrated by the remarkable fact that the nonlinear corrections cancel out for static BH in the full HS theory, reducing HS nonlinear equations to the free ones. This property is a HS analogue of the fact that the usual Kerr–Schild Ansatz reduces nonlinear Einstein equations to free Pauli-Fierz ones for a $4d$ BH. Let us stress that massless HS fields do not satisfy HS field equations in the BH background unless the HS interactions are switched on via the terms bilinear in the HS connections in the nonAbelian HS curvatures on the left hand side of (3.1).

In the proposed construction, static HS BH is described in terms of a Fock vacuum in the star–product algebra in the auxiliary twistor space. This Ansatz effectively projects $4d$ HS equations to $3d$ HS equations of [10] that describe $3d$ massive matter fields and can be solved with the aid of Wigner's deformed oscillators. The BH mass M coincides with the vacuum value $B_0 = \nu$ of [10] that sets the mass scale of $3d$ interacting massive fields. This reduction suggests an interesting duality between AdS_3 massive fields of a mass scale ν and $4d$ HS BH of mass M, $\nu = \lambda G M$, where $-\lambda^2$ is the cosmological constant and G is the Newton constant. More generally, the obtained results indicate that near BH fluctuations in HS theory describe a HS theory in the lower dimension thus providing a nontrivial dimensional compactification mechanism analogous to the brane picture in String Theory.

At the linearized level, where other fields do not contribute to the metric, the obtained solution reproduces AdS_4 Schwarzschild BH in the $s = 2$ similarly to the charged Reissner–Nordström BH that involves second order contribution of the electromagnetic field via its stress tensor. That the obtained solution exhibits leftover SUSY strongly indicates that it should be extremal. We believe that it can be further generalized to the NUT case using the phase ambiguity of $4d$ nonlinear HS equations of [2] as well as to the Kerr BH, extremal and not. More generally, a generalization to d dimensions is a challenging issue both at the free and at the nonlinear level. The preliminary analysis indicates that our method should work in various dimensions. It looks especially promising in the context of black rings [27] that exist in $d \geq 5$.

An exciting problem for the future study is to explore physical properties of the obtained solution. Here, the key question is how do small fluctuations propagate in the HS BH background? Its analysis should shed light on such fundamental concepts in HS BH physics as horizons, trapped surfaces *etc.* Since HS theory is essentially nonlocal at the interaction level, involving higher derivatives for higher spins, the analysis of these issues should be done by new means beyond standard GR machinery. For example, in HS theory it is not granted that the geodesic motion has much to do with signal propagation in the strong field regime. Moreover, in the HS theory with unbroken HS symmetries it is not even clear how to define a metric tensor beyond the linearized approximation. The study of thermodynamical interpretation of the HS BH is also of primary importance. We plan to consider these problems in the future.

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Appendix. Conventions

For any Lorentz vector ξ^n, its two-component spinor counterpart is $\xi_{\alpha\dot{\alpha}} = \xi^n \sigma_{n,\alpha\dot{\alpha}}$, where $\sigma_{n,\alpha\dot{\alpha}} = (I_{\alpha\dot{\alpha}}, \sigma_{i,\alpha\dot{\alpha}})$ contains unity matrix I along with Pauli matrices. Latin indices are raised and lowered by Minkowski metric η_{mn}. Spinorial indices are raised and lowered according to

$$A_\alpha = A^\beta \epsilon_{\beta\alpha}\,, \qquad A^\alpha = A_\beta \epsilon^{\alpha\beta}\,, \qquad \bar{A}_{\dot{\alpha}} = \bar{A}^{\dot{\beta}} \epsilon_{\dot{\beta}\dot{\alpha}}\,, \qquad \bar{A}^{\dot{\alpha}} = \bar{A}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}}\,. \tag{A.1}$$

$Sp(4)$ indices $A = (\alpha, \dot{\alpha}) = 1 \dots 4$ are raised and lowered by the canonical symplectic form

$$\epsilon_{AB} = -\epsilon_{BA} = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}, \qquad A_A = A^B \epsilon_{BA}\,, \qquad A^A = A_B \epsilon^{AB}\,. \tag{A.2}$$

To distinguish between two types of projectors $\Pi_{\pm AB}$ and $\pi^{\pm}_{\alpha\beta}$ we use the convention with lower and upper labels $\pm$ assigned to the objects projected by $\Pi_{\pm AB}$ and by $\pi^{\pm}_{\alpha\beta}$, respectively. For example,

$$Y_{\pm A} = \Pi_{\pm A}{}^B Y_B\,, \quad y_{\pm\alpha} = \Pi_{\pm\alpha}{}^B Y_B\,, \quad \bar{y}_{\pm\dot{\alpha}} = \Pi_{\pm\dot{\alpha}}{}^B Y_B\,, \tag{A.3}$$

but

$$y^{\pm}_{\alpha} = \pi^{\pm\beta}_{\alpha} y_{\beta}\,, \qquad \bar{y}^{\pm}_{\dot{\alpha}} = \bar{\pi}^{\pm\dot{\beta}}_{\dot{\alpha}} \bar{y}_{\dot{\beta}}\,. \tag{A.4}$$

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